

Chapter 3.10

Analysis Techniques

Maximum Power Transfer

Engr228 - Circuit Analysis
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Section 3.10 Objective

- Learn to establish the conditions for maximum transfer of current, voltage, or power from an input circuit to an external load.

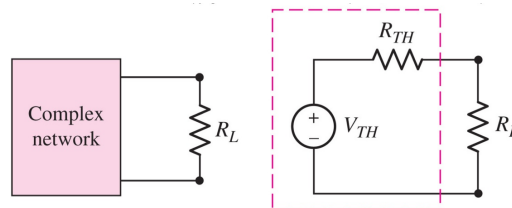
Finding the Thévenin Equivalent

- Disconnect the load;
- Find the open circuit voltage v_{oc} ;
- Find the equivalent resistance R_{eq} of the network with all independent sources turned off.
 - Set voltage sources to zero volts → short circuit
 - Set current sources to zero amps → open circuit

Then:

$$V_{TH} = v_{oc} \text{ and}$$

$$R_{TH} = R_{eq}$$

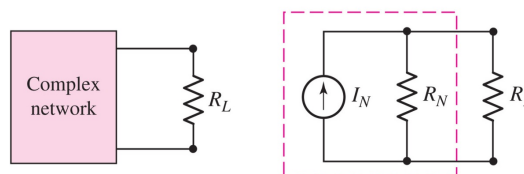


Finding the Norton Equivalent

- Replace the load with a short circuit;
- Find the short circuit current i_{sc} ;
- Find the equivalent resistance R_{eq} of the network with all independent sources turned off (same as Thévenin)
 - Set voltage sources to zero volts → short circuit;
 - Set current sources to zero amps → open circuit.

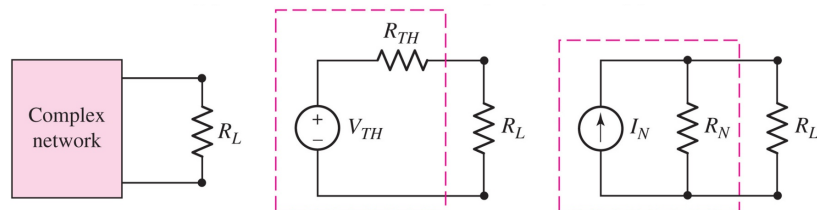
Then:

$$I_N = i_{sc} \text{ and } R_N = R_{eq}$$



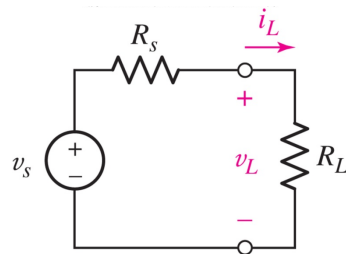
Source Transformation: Norton and Thévenin

The Thévenin and Norton equivalents are source transformations of each other.

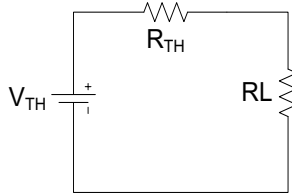


Maximum Power Transfer

Thévenin's or Norton's equivalent circuit delivers a maximum power to the load R_L for which $R_{TH} = R_L$



Maximum Power Theorem Proof



$$P = I^2 R_L \quad \text{and} \quad I = \frac{V_{TH}}{R_{TH} + R_L}$$

$$\text{Plug it in } P = \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 \cdot R_L = \frac{V_{TH}^2 R_L}{(R_{TH} + R_L)^2}$$

$$\frac{dP}{dR_L} = \frac{(R_{TH} + R_L)^2 V_{TH}^2 - V_{TH}^2 R_L \cdot 2(R_{TH} + R_L)}{(R_{TH} + R_L)^4} = 0$$

Maximum Power Theorem Proof - continued

$$\frac{dP}{dR_L} = \frac{(R_{TH} + R_L)^2 V_{TH}^2 - V_{TH}^2 R_L \cdot 2(R_{TH} + R_L)}{(R_{TH} + R_L)^4} = 0$$

$$(R_{TH} + R_L)^2 V_{TH}^2 = V_{TH}^2 R_L \cdot 2(R_{TH} + R_L)$$

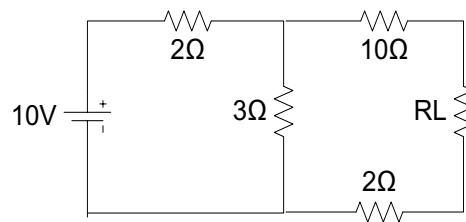
$$(R_{TH} + R_L) = 2R_L$$

$$R_{TH} = R_L$$

For maximum power transfer

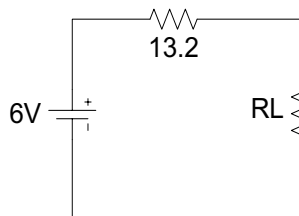
Maximum Power Example

Evaluate R_L for maximum power transfer and find the power.



Maximum Power Example - continued

Thévenin's equivalent circuit



R_L should be set to 13.2Ω to get maximum power transfer.

$$\text{Max. power is } \frac{V^2}{R} = \frac{(6/2)^2}{13.2} = 0.68W$$

Section 3.10 Summary

- You learned to establish the conditions for maximum transfer of current, voltage, or power from an input circuit to an external load.